Microlens quality assessment using the Extended Nijboer-Zernike diffraction theory

J.J.M. Braat, S. van Haver, S.F. Pereira

Delft University of Technology
Department of Imaging Science and Technology
Optics Research Group
Lorentzweg 1, 2628 CJ Delft, The Netherlands

http://www.nijboerzernike.nl

E-mail: j.j.m.braat@tudelft.nl

Contents

- General approach to lens quality measurement
- Through-focus intensity measurements and aberration retrieval
- Extended Nijboer-Zernike theory of diffraction using the Debye integral
- The Rayleigh and Debye diffraction integral, small-numerical-aperture case
- Required modifications of the Debye integral
- Numerical tests and error analysis
- Conclusions

Lens quality assessment

- Methods:

  interferometry, **point-spread function** analysis,
  Shack-Hartmann test, frequency transfer etc.

- Point-spread function analysis from through-focus intensity plots
  - Gerchberg-Saxton algorithm (pupil – image plane)
  - frequency transfer recovery (electron microscopy)
  - **Extended Nijboer-Zernike (ENZ) aberration retrieval**

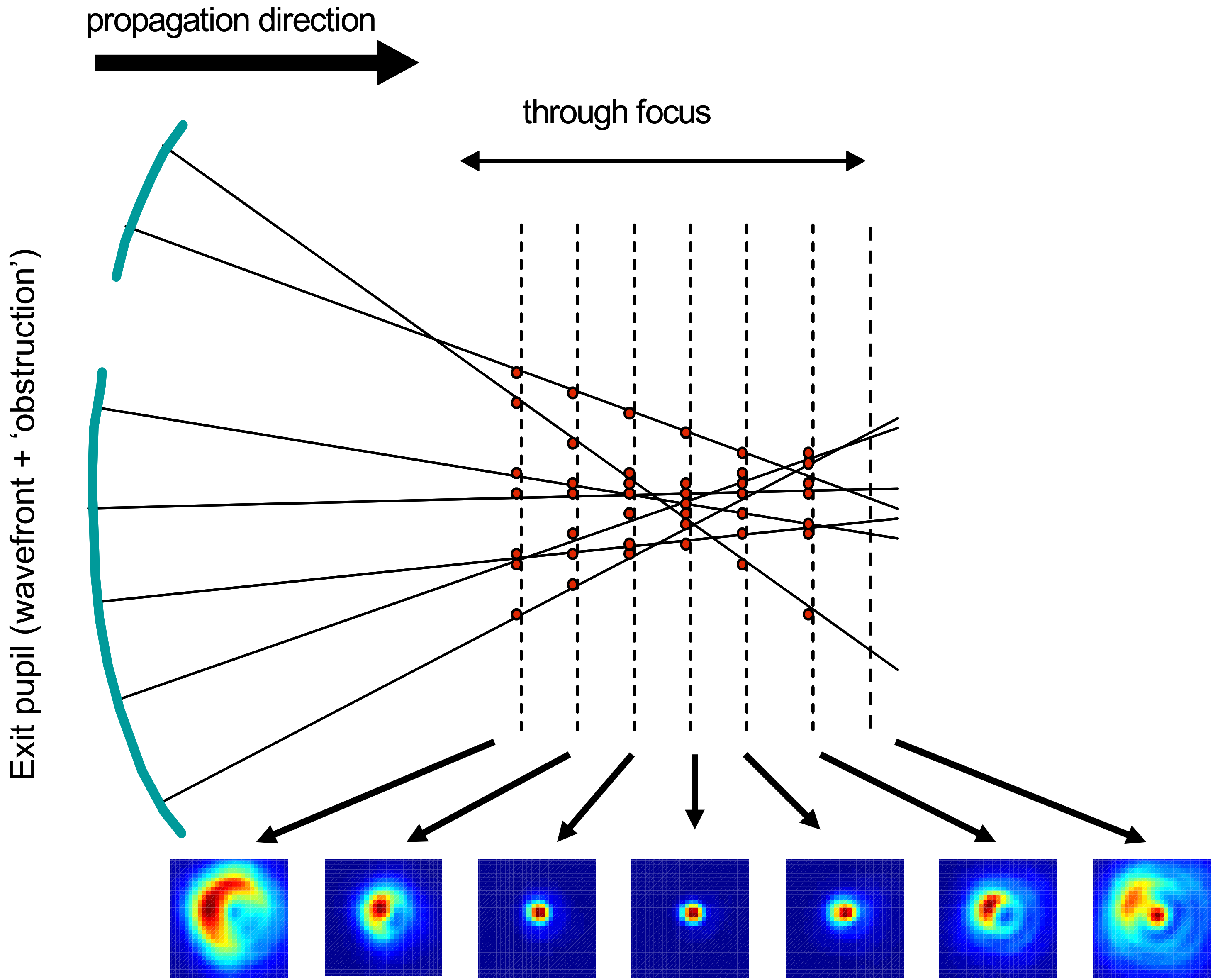
↓

- **ENZ-retrieval** based on Debye diffraction integral for point-spread formation. For small NA and/or small pupil diameter, Rayleigh diffraction integral is required!

Contents

- General approach to lens quality measurement
- Through-focus intensity measurements and aberration retrieval
- Extended Nijboer-Zernike theory of diffraction using the Debye integral
- The Rayleigh and Debye diffraction integral, small-numerical-aperture case
- Required modifications of the Debye integral
- Numerical tests and error analysis
- Conclusions

Phase retrieval from through-focus intensity patterns

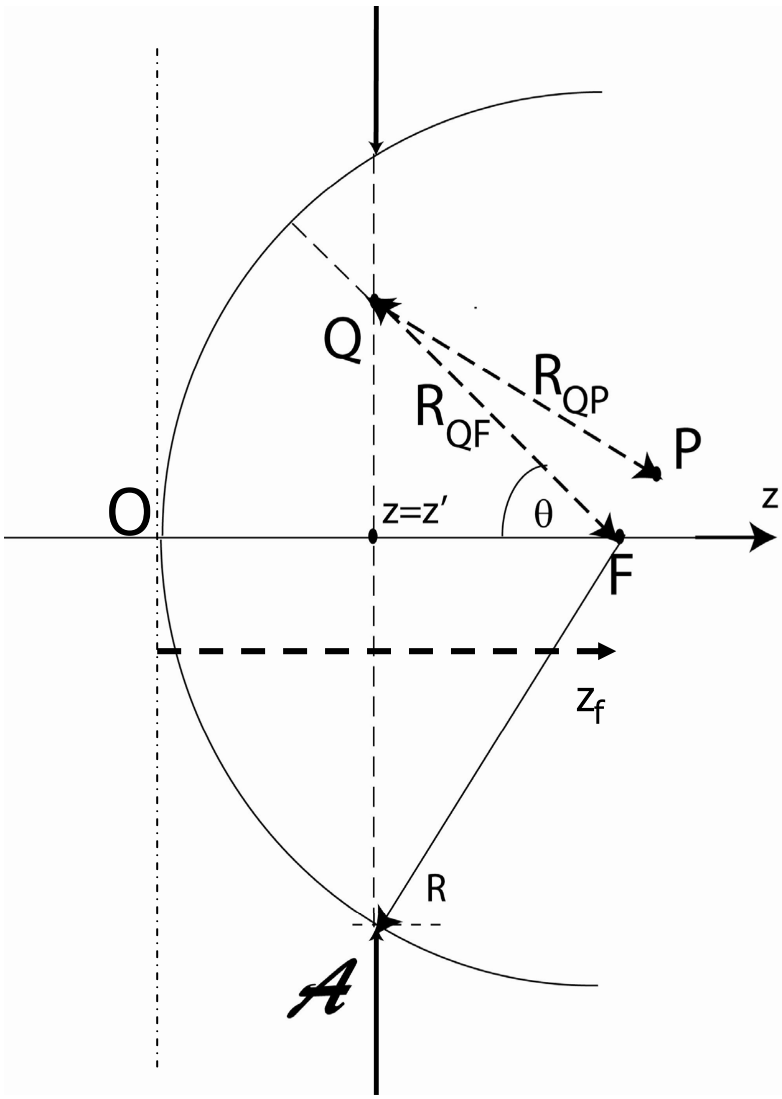


Diffraction geometry (exit pupil → image)

$\mathcal{A}$ is (image of) diffracting aperture

R is distance exit pupil to focus F

- Debye integral is an approximation of the field in a general point P if $R \rightarrow \infty$
- Integration is over all points Q in the diffracting aperture plane $\mathcal{A}$ or over the exit pupil sphere through the origin O.
- Defocusing range $\Delta << R$



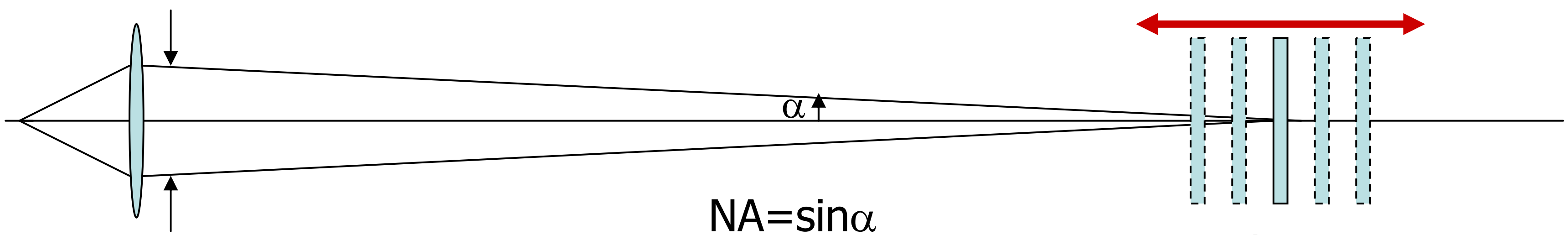
CCD detection of through-focus images

- For a slightly aberrated lens system, the almost diffraction-limited spot should be well resolved by the detector pixels

- 10 pixels (5 μm each) for a typical diffraction unit of 0.6 λ/NA

- With λ=633 nm, NA choice in range $0.005<NA<0.01$

- Detection of source point at low aperture conjugate side!



Contents

- General approach to lens quality measurement
- Through-focus intensity measurements and aberration retrieval
- Extended Nijboer-Zernike theory of diffraction using the Debye integral
- The Rayleigh and Debye diffraction integral, small-numerical-aperture case
- Required modifications of the Debye integral
- Numerical tests and error analysis
- Conclusions

The ENZ-formalism based on the Debye integral

$$U(r,\phi;f)=\frac{1}{\pi}\int_0^1\int_0^{2\pi}\exp\{if\rho^2\}\boxed{P(\rho,\theta)}\exp\{2\pi i\rho r\cos(\theta-\phi)\}\,\rho d\rho d\theta$$

$$P(\rho,\theta)=A(\rho,\theta)\exp\{i\Phi(\rho,\theta)\}=\sum_{m,n}\beta_n^m R_n^m(\rho)\exp\{im\theta\}$$

$$U(r,\phi;f)=2\sum i^m\beta_n^m V_n^m(r,f)\exp\{im\phi\}$$

$$V_n^m(r,f)=\exp(if)\sum_{l=1}^{\infty}(-2if)^{l-1}\sum_{j=0}^{p}v_{lj}\frac{J_{m+l+2j}(2\pi r)}{l(2\pi r)^l}$$

$$v_{lj}=(-1)^p\,(m+l+2j)\binom{m+j+l-1}{l-1}\binom{j+l-1}{l-1}\binom{l-1}{p-j}\Big/\binom{q+l+j}{l}$$

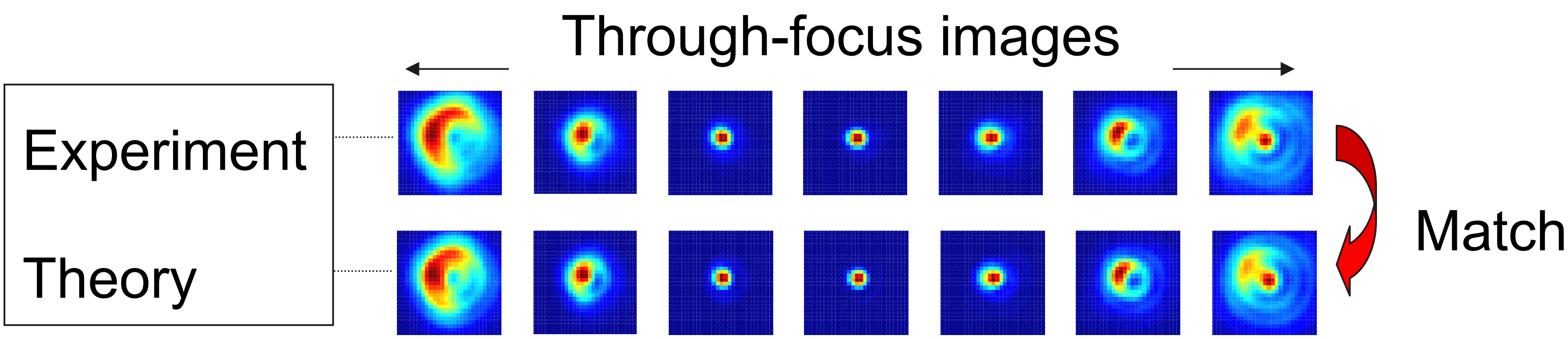
Detection of $I(r,\phi,f)=|U(r,\phi,f)|^2$, nonlinear operation !

Linearized retrieval method for Zernike coefficients

Basic principle

Observed intensity	= analytic expression
	≈ linearized analytic expression
	= $\sum \beta(m,n)$ x basic-functions

Match experiment to theory by solving a linear system of equations in $\beta(m,n)$ with the aid of recorded images



Contents

- General approach to lens quality measurement
- Through-focus intensity measurements and aberration retrieval
- Extended Nijboer-Zernike theory of diffraction using the Debye integral
- The Rayleigh and Debye diffraction integral, small-numerical-aperture case
- Required modifications of the Debye integral
- Numerical tests and error analysis
- Conclusions

Rayleigh and Debye diffraction integral; small-NA expressions for axial intensity

- Analytic expression for Rayleigh on-axis intensity given by H. Osterberg and L.W. Smith, J. Opt. Soc. Am. **51**, 1050-1054 (1961)
- Concept of optical axial coordinate introduced by Y. Li, J. Opt. Soc. Am. **72**, 770–774 (1982). Further refined by C.J.R. Sheppard and P. Török, J. Opt. Soc. Am. A **20**, 2156-2162 (2003).

$$\text{Optical axial coordinate (small NA)}: \quad f = \frac{kR\Delta \sin^2\alpha}{2(R+\Delta)}.$$

$$\text{Rayleigh on-axis intensity (small NA)}:$$

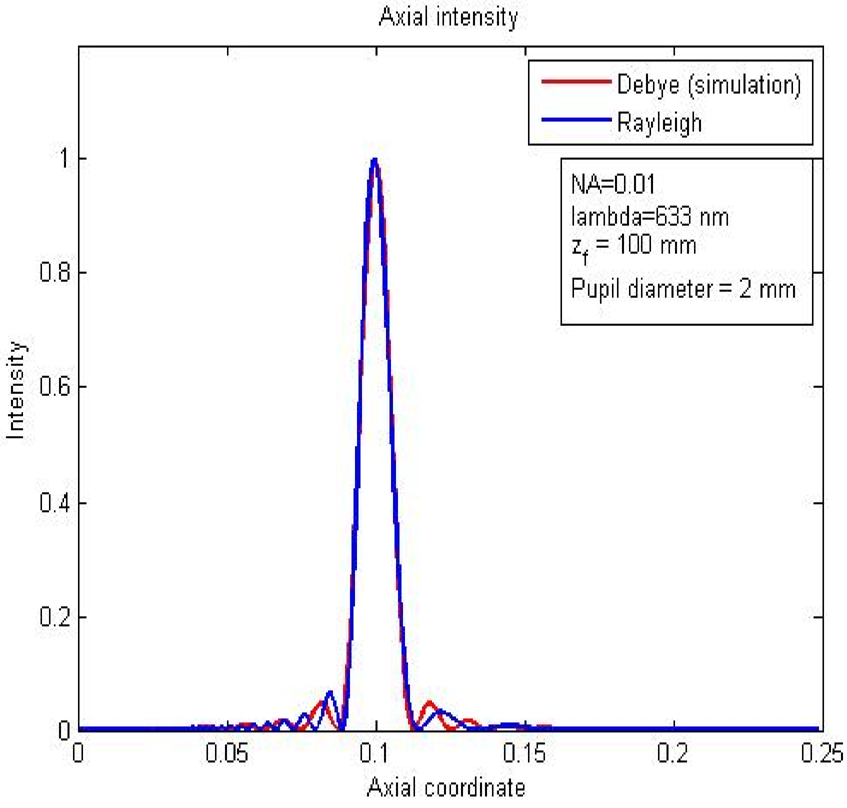
$$I(\Delta) = \left(\frac{f}{\Delta}\right)^2 \operatorname{sinc}^2\left(\frac{f}{2}\right).$$

$$\text{Debye approximation for optical axial coordinate}: \quad f \approx \frac{k\Delta \sin^2\alpha}{2}.$$

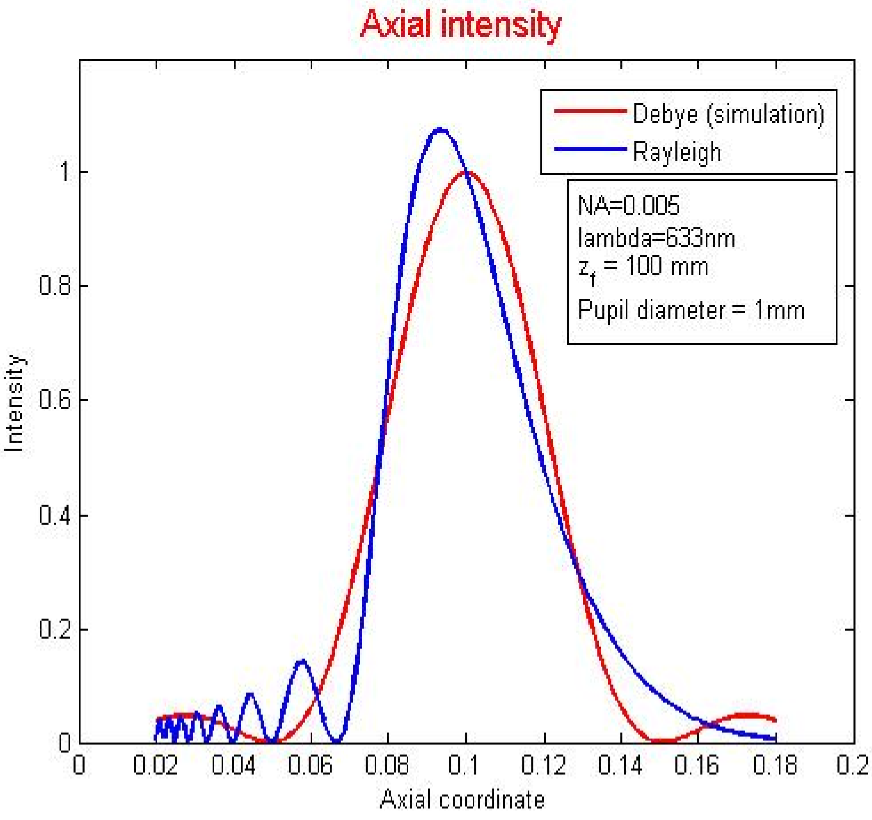
$$\text{Debye on-axis intensity} \ \ (\Delta / R \to 0):$$

$$I(\Delta) = \left[\frac{k\sin^2\alpha}{2}\right]^2 \operatorname{sinc}^2\left(\frac{k\Delta\sin^2\alpha}{4}\right)^2.$$

Rayleigh and Debye diffraction integral, small NA features



NA=0.01



NA=0.005

Contents

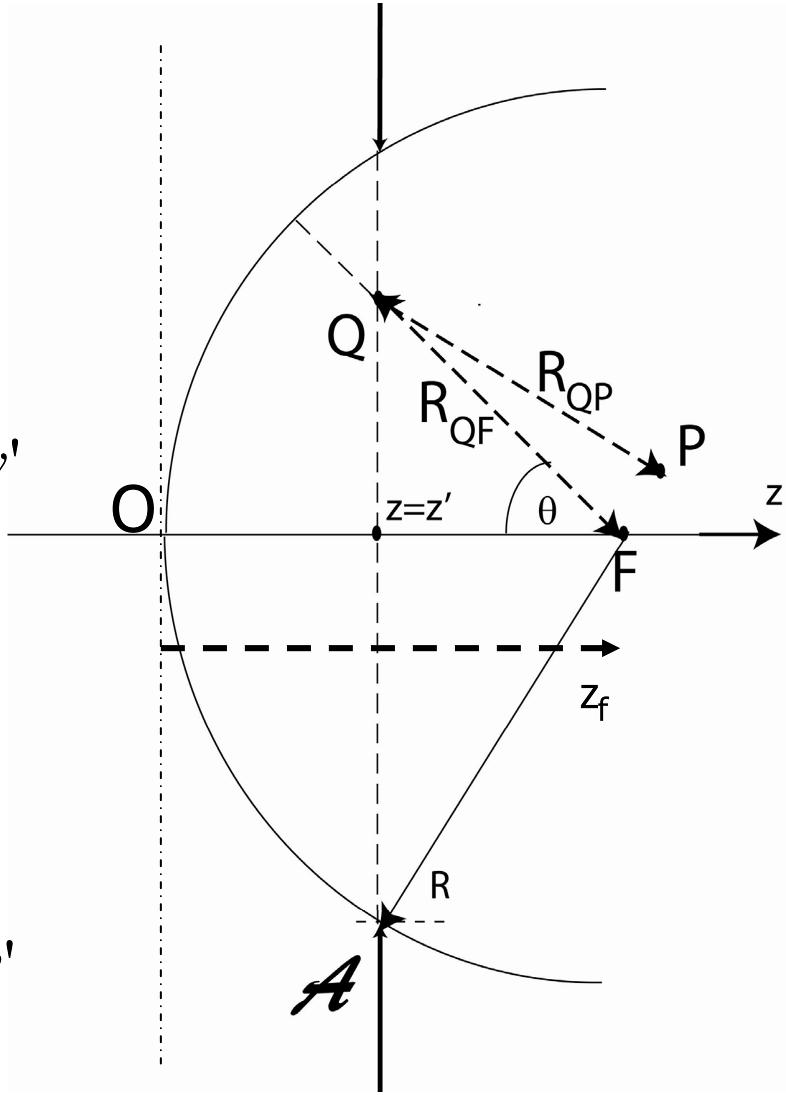
Required modification of the Debye integral to approximate the Rayleigh integral: axial *and* lateral coordinate transformation

Rayleigh

$$U_R(x,y,z) = \frac{-i}{\lambda} \iint_{\mathcal{A}} \frac{z-z'}{R_{QP}^2 R_{QF}} U_0(x',y') \times \exp\{ik(R_{QP}-R_{QF})\} dx' dy'$$

Debye

$$U_D(x,y,z) = \frac{-i}{\lambda} \iint_{\mathcal{A}} \frac{z_f - z'}{R_{QF}^3} U_0(x',y') \times \exp\{i\vec{k} \cdot (\vec{r}_{QP} - \vec{r}_{QF})\} dx' dy'$$



Debye → Rayleigh: axial and lateral coordinate transformation

Debye integral in normalized coordinates :

$$U_D(x,y,z) = \frac{-i\sin^2\alpha}{\lambda} \iint_{\text{unit circle}} U_p(\rho,\vartheta)\exp\{i\,\mathbf{f}\rho^2\}\exp\{-i2\pi\,\mathbf{r}\rho\cos(\vartheta-\varphi)\}\rho\, d\rho\, d\vartheta.$$

$$\exp\{i\,f\rho^2\} \quad\rightarrow\quad \exp\left\{i\left(\frac{f}{1+\dfrac{f\lambda}{\pi R\sin^2\alpha}}\right)\rho^2\right\} = \exp\left(i\,f'\rho^2\right)\left(\begin{array}{c}\text{compaction of distance}\\ \text{between axial intensity}\\ \text{zeros towards aperture}\end{array}\right).$$

$$\exp\{-i2\pi\,r\rho\cos(\vartheta-\varphi)\} \quad\rightarrow\quad \exp\{-i2\pi\,r'\rho\cos(\vartheta-\varphi)\}\ \text{with}$$

$$r' = r\left(\frac{1}{1+\dfrac{f\lambda}{\pi R\sin^2\alpha}}\right)\left\{1-\frac{\lambda^2}{4\pi^2R^2\sin^2\alpha}\left[1-\frac{2f\lambda}{\pi R\sin^2\alpha}\right]r^2\right\}\left(\begin{array}{c}\text{lateral expansion diffraction}\\ \text{pattern as a function of}\\ \text{distance from aperture}\end{array}\right).$$

Normalized amplitude correction factor : $\dfrac{1}{1+\dfrac{f\lambda}{\pi R\sin^2\alpha}}$

Debye → Rayleigh:
region of applicability of the coordinate transformation

$$U_D(x,y,z) = \frac{-i\sin^2\alpha}{\lambda} \iint_{\text{unit circle}} U_p(\rho,\vartheta)\exp\{i\,\mathbf{f}\rho^2\}\exp\{-i2\pi\,\mathbf{r}\rho\cos(\vartheta-\varphi)\}\rho\, d\rho\, d\vartheta.$$

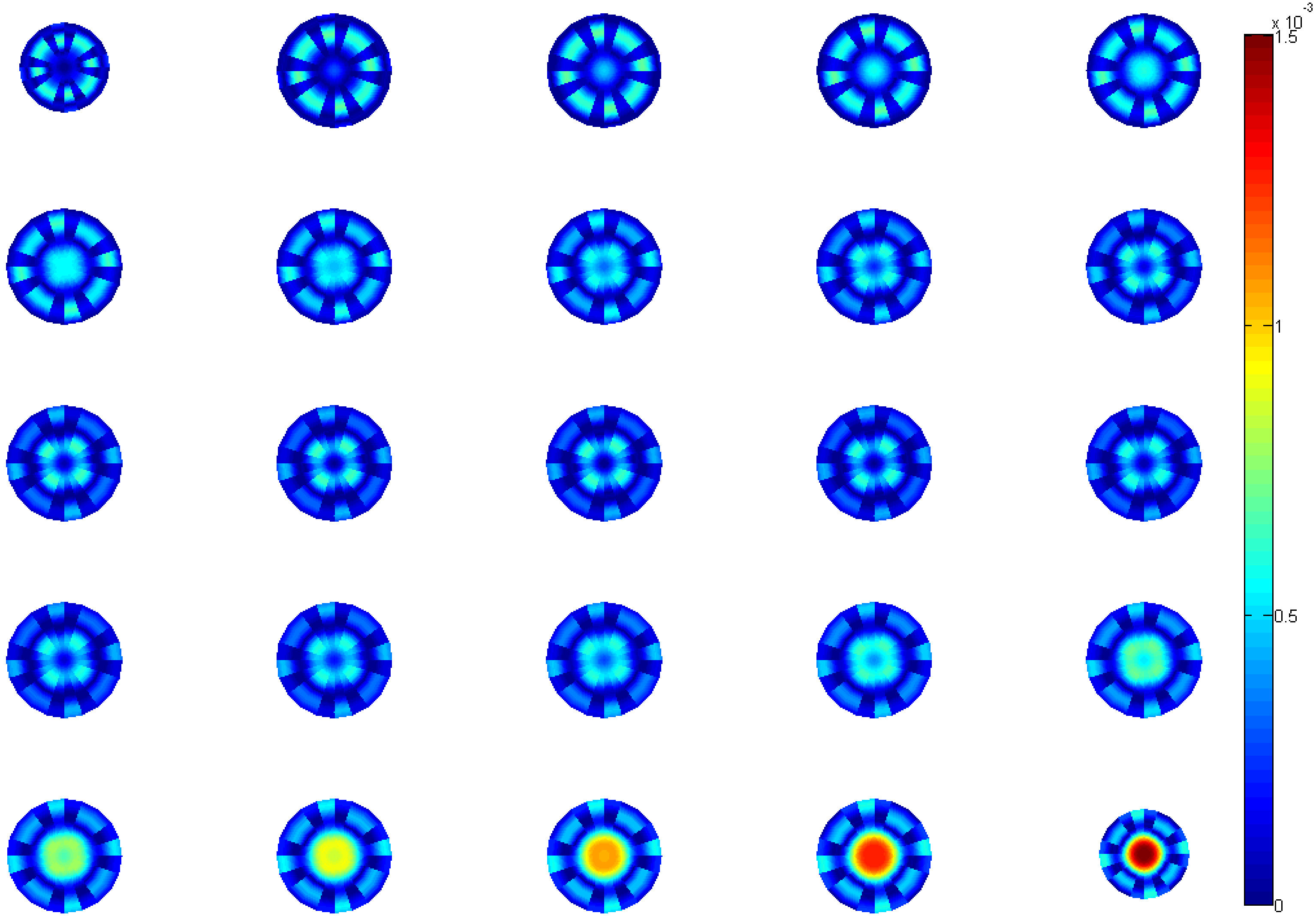
In adapting the Debye integral to the Rayleigh integral by an axial and lateral coordinate transformation ($r \to r'$, $f \to f'$), higher order terms in the exponential that are proportional to $r^2\rho^2$ and $r\rho^3$ could not be included.

With the requirement that these exponential factors nowhere exceed the value of 0.1 radian, we find the following region of applicability of the coordinate transformation

a) pupil diameter $\quad 2\rho_0 \geq 60\lambda$

b) axial distance exit pupil to focus $\quad R \geq 300\lambda$

Debye → Rayleigh: residual errors after transformation



Contents

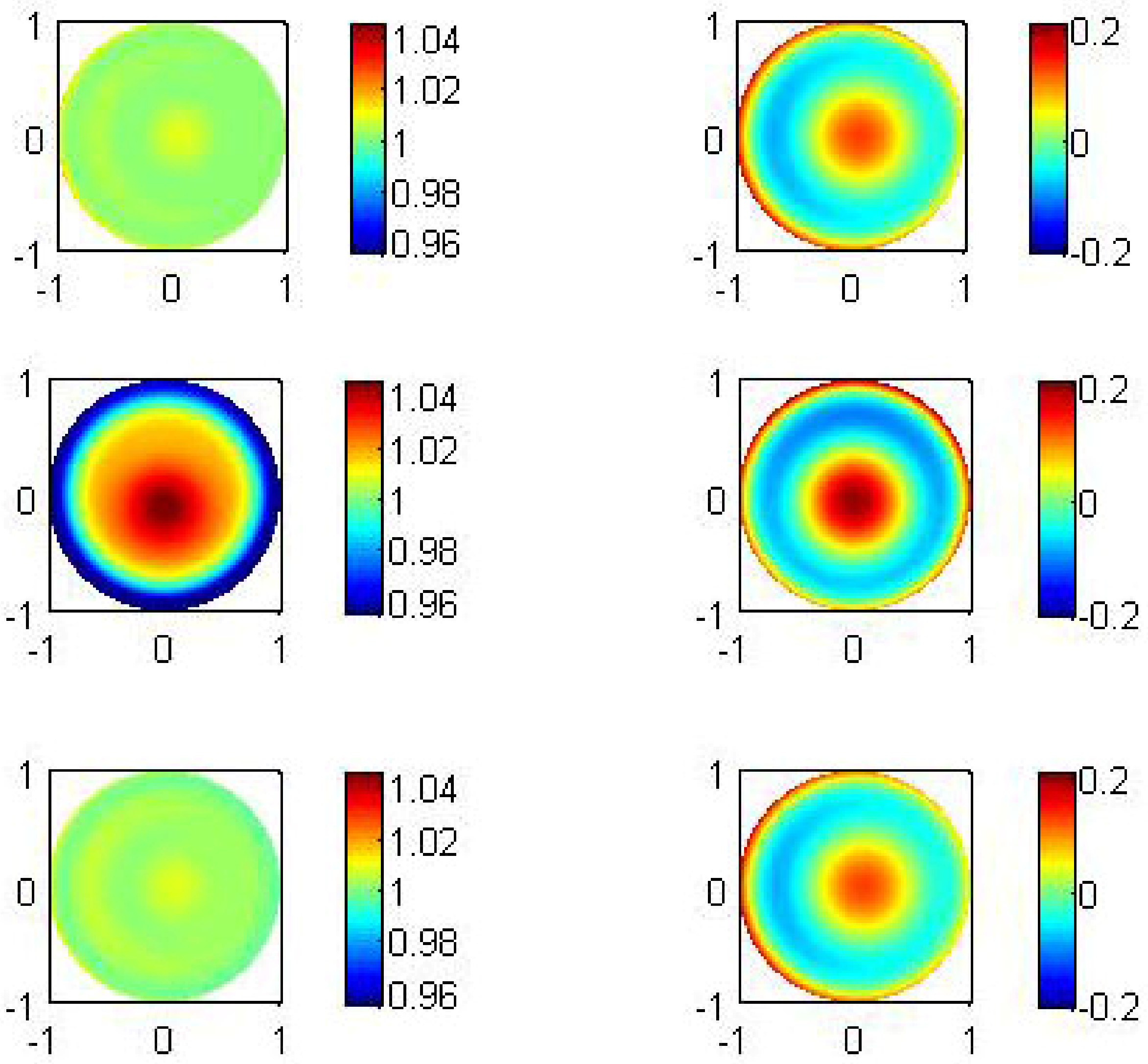
- General approach to lens quality measurement
- Through-focus intensity measurements and aberration retrieval
- Extended Nijboer-Zernike theory of diffraction using the Debye integral
- The Rayleigh and Debye diffraction integral, small-numerical-aperture case
- Required modifications of the Debye integral
- Numerical tests and error analysis
- Conclusions

Pupil function retrieval based on adapted Debye integral

Input pupil function (amplitude / phase)

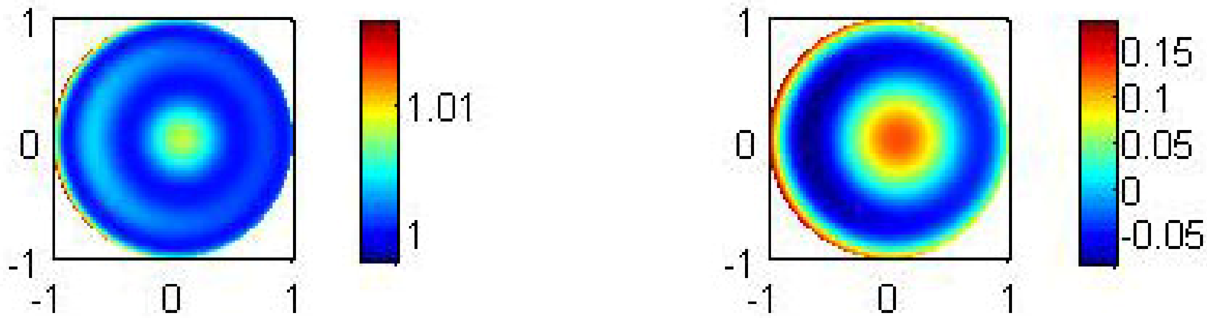
Standard retrieval (Debye integral)

Retrieval using adap-ted Debye integral

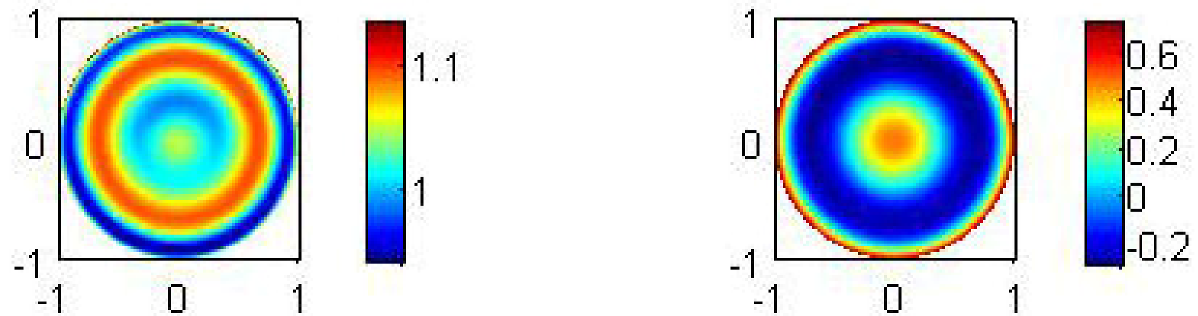


Microlens pupil function retrieval

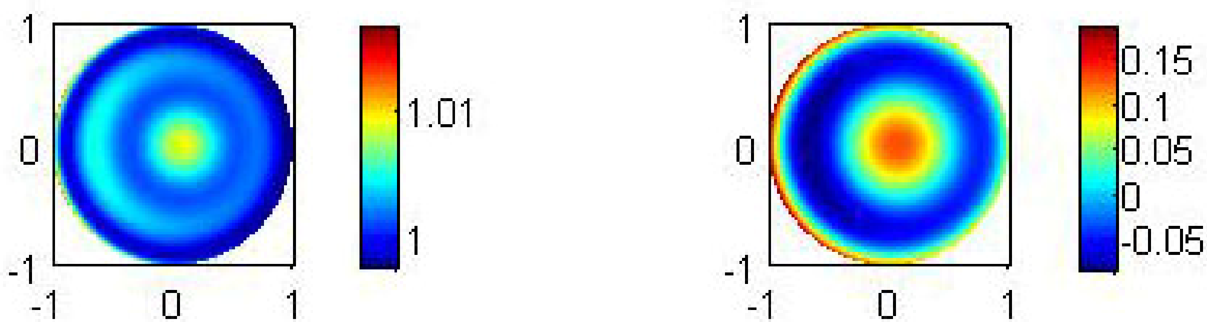
Microlens pupil function
(amplitude / phase)



Standard retrieval
(Debye integral)



Retrieval using adap-
ted Debye integral



Pupil function retrieval, numerical data

NA = 0.005, diameter pupil = 0.1 mm, λ=266 nm

Input pupil function (complex Zernike coefficients)

$\beta_0^0 = 1.000$ $\beta_2^0 = 0.000$ $\beta_4^0 = 0.1230i$

$\beta_3^1 = 0.0300i$ $\beta_3^{-1} = 0.0300i$

Standard retrieval with Debye integral

$\beta_0^0 = 0.9942$ $\beta_2^0 = -0.0481 - 0.0285i$ $\beta_4^0 = -0.0184 + 0.1719i$

$\beta_3^1 = -0.0284 + 0.0053i$ $\beta_3^{-1} = 0.0284 - 0.0053i$

Retrieval with adapted Debye integral

$\beta_0^0 = 1.0001$ $\beta_2^0 = -0.0020$ $\beta_4^0 = -0.0025 + 0.1227i$

$\beta_3^1 = 0.0299i$ $\beta_3^{-1} = 0.0299i$

Microlens pupil function retrieval, numerical data

NA = 0.01, diameter pupil = 0.04 mm, λ=400 nm

Input pupil function (complex Zernike coefficients)

$\beta_0^0 = 1.000$ $\beta_2^0 = 0.000$ $\beta_4^0 = 0.1230i$

$\beta_3^1 = 0.0300i$ $\beta_3^{-1} = 0.0300i$

Standard retrieval with Debye integral

$\beta_0^0 = 0.9931$ $\beta_2^0 = -0.0516 + 0.1161i$ $\beta_4^0 = -0.1119 + 0.5887i$

$\beta_3^1 = -0.0244 + 0.0087i$ $\beta_3^{-1} = -0.0244 + 0.0087i$

Retrieval with adapted Debye integral

$\beta_0^0 = 1.0001$ $\beta_2^0 = -0.0022 + 0.0003i$ $\beta_4^0 = -0.0017 + 0.1259i$

$\beta_3^1 = 0.0299i$ $\beta_3^{-1} = 0.0299i$

Conclusion

- Small-NA imaging (NA<0.01) introduces asymmetry around focus
- The ENZ pupil function retrieval method based on the Debye integral has to be adapted by an axial and lateral coordinate transformation that scales with the defocusing ($NA \leq 0.01$)
- The region of applicability of the adapted retrieval method is adequate for microscope objectives and microlenses provided the physical diameter of the lens aperture exceeds typically 50 wavelengths
- Numerical experiments showed the validity of the adapted ENZ retrieval method for small NA systems and microlenses