

Micro lens quality assessment using the Extended Nijboer-Zernike diffraction theory

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Contents

- General approach to lens quality measurement
- Through-focus intensity measurements and aberration retrieval
- Extended Nijboer-Zernike theory of diffraction using the Debye integral
- The Rayleigh and Debye diffraction integral, small-numerical-aperture case
- Required modifications of the Debye integral
- Numerical tests and error analysis
- Conclusions

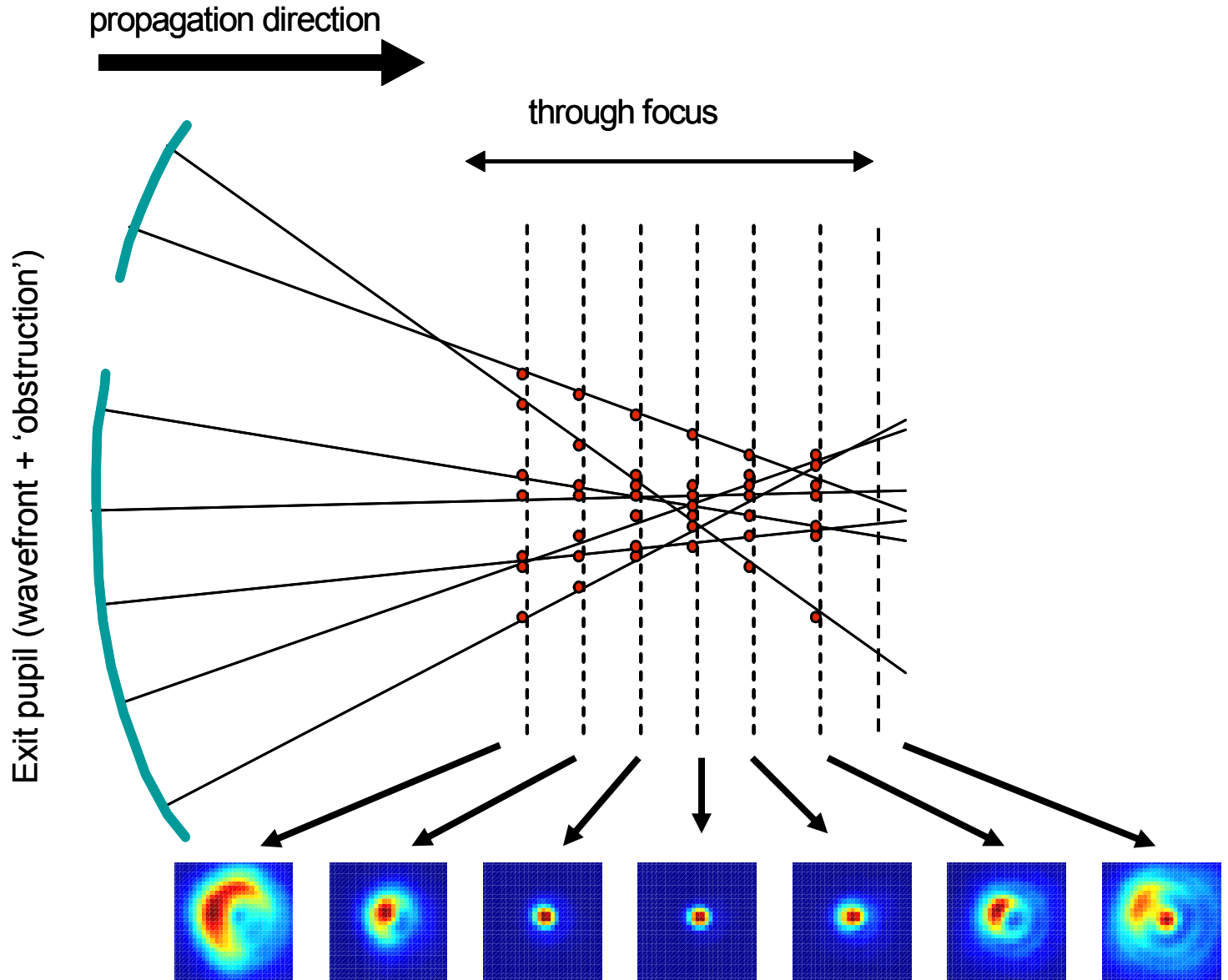
Lens quality assessment

- Methods:
interferometry, **point-spread function analysis**,
Shack-Hartmann test, frequency transfer etc.
 - Point-spread function analysis from through-focus intensity plots
 - Gerchberg-Saxton algorithm (pupil – image plane)
 - frequency transfer recovery (electron microscopy)
 - **Extended Nijboer-Zernike (ENZ) aberration retrieval**
- ↓
- **ENZ-retrieval** based on Debye diffraction integral for point-spread formation. For small NA and/or small pupil diameter, Rayleigh diffraction integral is required!

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Phase retrieval from through-focus intensity patterns

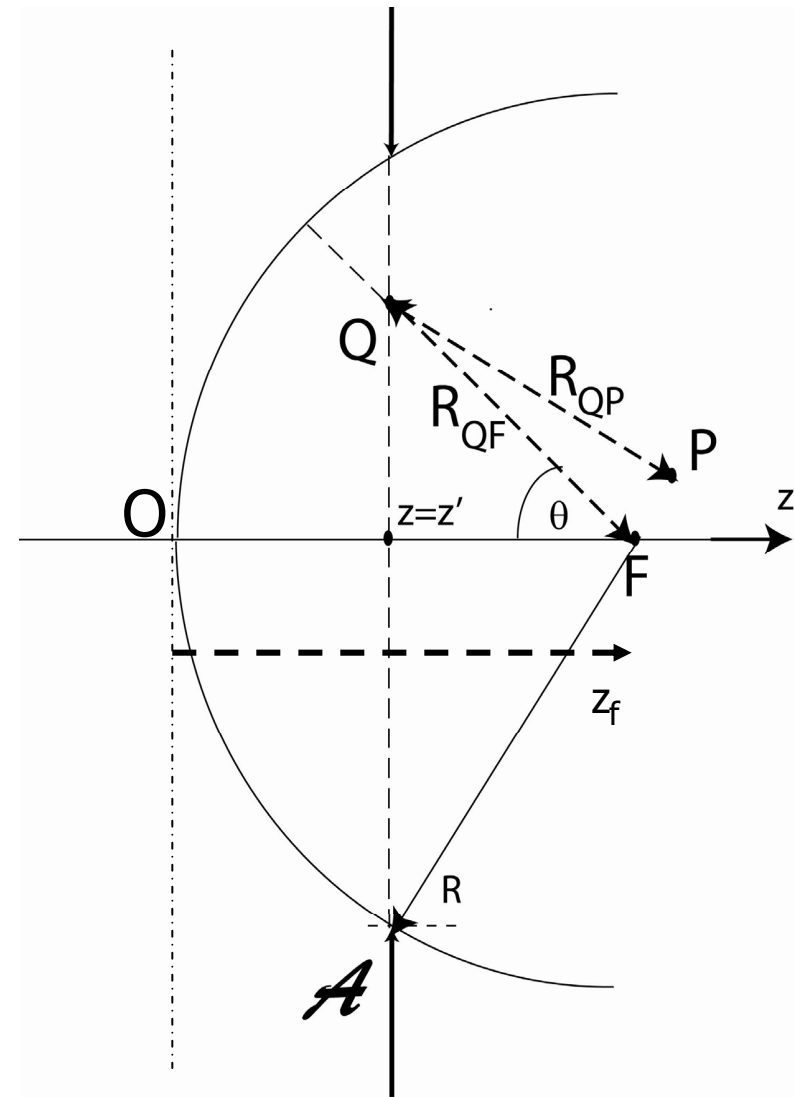


Diffraction geometry (exit pupil \rightarrow image)

\mathcal{A} is (image of) diffracting aperture

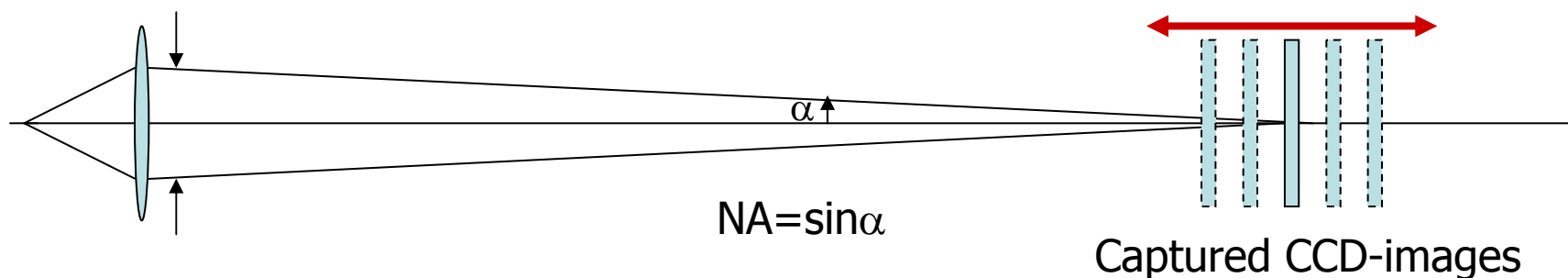
R is distance exit pupil to focus F

- Debye integral is an approximation of the field in a general point P if $R \rightarrow \infty$
- Integration is over all points Q in the diffracting aperture plane \mathcal{A} or over the exit pupil sphere through the origin O .
- Defocusing range $\Delta \ll R$



CCD detection of through-focus images

- For a slightly aberrated lens system, the almost diffraction-limited spot should be well resolved by the detector pixels
- 10 pixels (5 μm each) for a typical diffraction unit of $0.6 \lambda/\text{NA}$
- With $\lambda=633 \text{ nm}$, NA choice in range $0.005 < \text{NA} < 0.01$
- Detection of source point at low aperture conjugate side!



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The ENZ-formalism based on the Debye integral

$$U(r, \phi; f) = \frac{1}{\pi} \int_0^1 \int_0^{2\pi} \exp\{if\rho^2\} P(\rho, \theta) \exp\{2\pi i\rho r \cos(\theta - \phi)\} \rho d\rho d\theta$$

$$P(\rho, \theta) = A(\rho, \theta) \exp\{i\Phi(\rho, \theta)\} = \sum_{m,n} \beta_n^m R_n^m(\rho) \exp\{im\theta\}$$

$$U(r, \phi; f) = 2 \sum i^m \beta_n^m V_n^m(r, f) \exp\{im\phi\}$$

$$V_n^m(r, f) = \exp(if) \sum_{l=1}^{\infty} (-2if)^{l-1} \sum_{j=0}^p v_{lj} \frac{J_{m+l+2j}(2\pi r)}{l(2\pi r)^l}$$

$$v_{lj} = (-1)^p (m+l+2j) \binom{m+j+l-1}{l-1} \binom{j+l-1}{l-1} \binom{l-1}{p-j} / \binom{q+l+j}{l}$$

Detection of $I(r, \phi, f) = |U(r, \phi, f)|^2$, nonlinear operation !

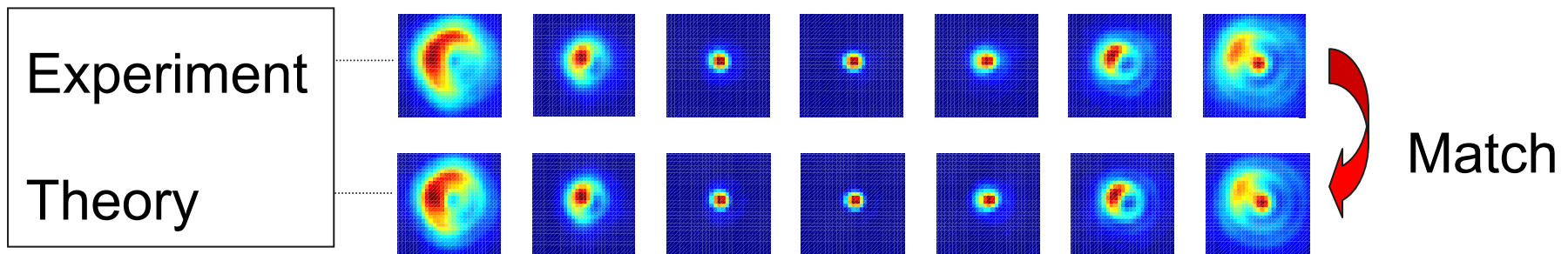
Linearized retrieval method for Zernike coefficients

Basic principle

$$\begin{aligned} \text{Observed intensity} &= \text{analytic expression} \\ &\approx \text{linearized analytic expression} \\ &= \sum \beta(m,n) \times \text{basic-functions} \end{aligned}$$

Match experiment to theory by solving a linear system of equations in $\beta(m,n)$ with the aid of recorded images

Through-focus images



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Rayleigh and Debye diffraction integral; small-NA expressions for axial intensity

- Analytic expression for Rayleigh on-axis intensity given by H. Osterberg and L.W. Smith, J. Opt. Soc. Am. **51**, 1050-1054 (1961)
- Concept of optical axial coordinate introduced by Y. Li, J. Opt. Soc. Am. **72**, 770–774 (1982). Further refined by C.J.R. Sheppard and P. Török, J. Opt. Soc. Am. A **20**, 2156-2162 (2003).

Optical axial coordinate (small NA): $f = \frac{kR\Delta \sin^2 \alpha}{2(R + \Delta)}$.

Rayleigh on - axis intensity (small NA):

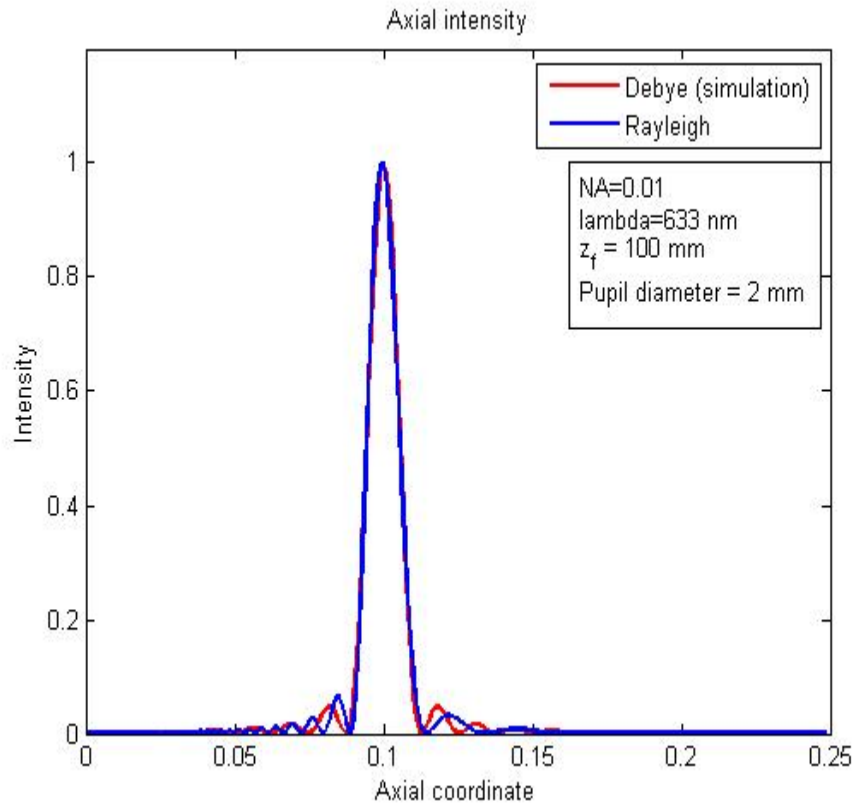
$$I(\Delta) = \left(\frac{f}{\Delta}\right)^2 \operatorname{sinc}^2\left(\frac{f}{2}\right).$$

Debye approximation for optical axial coordinate: $f \approx \frac{k\Delta \sin^2 \alpha}{2}$.

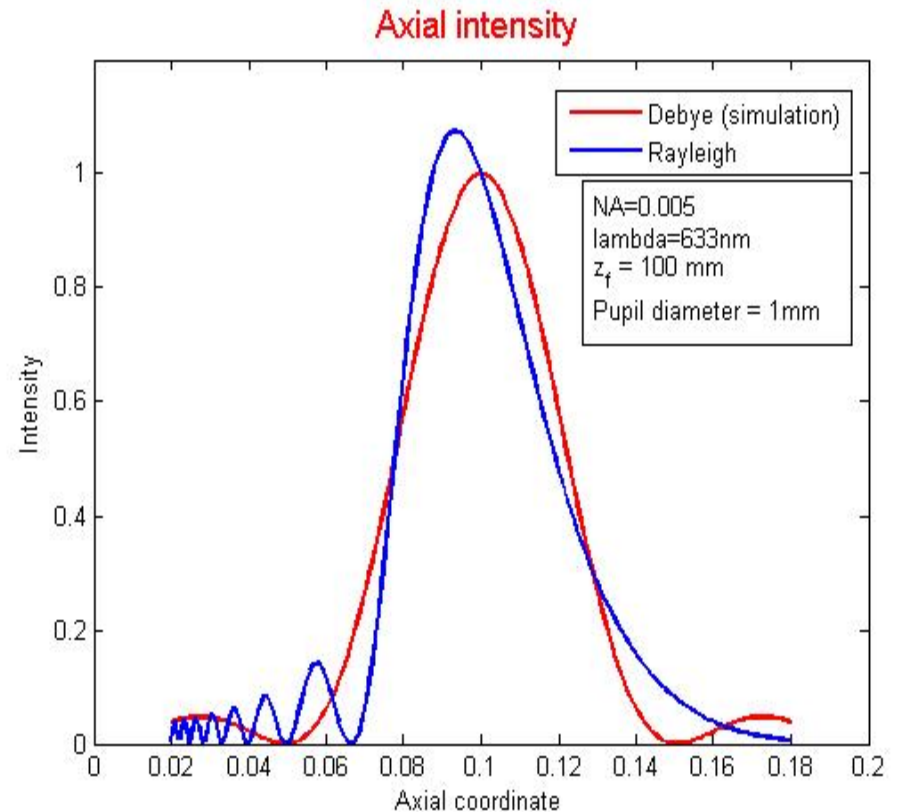
Debye on - axis intensity ($\Delta/R \rightarrow 0$):

$$I(\Delta) = \left[\frac{k \sin^2 \alpha}{2}\right]^2 \operatorname{sinc}^2\left(\frac{k\Delta \sin^2 \alpha}{4}\right).$$

Rayleigh and Debye diffraction integral, small NA features



NA=0.01



NA=0.005

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Debye → Rayleigh: axial and lateral coordinate transformation

Debye integral in normalized coordinates :

$$U_D(x, y, z) = \frac{-i \sin^2 \alpha}{\lambda} \iint_{\text{unit circle}} U_p(\rho, \vartheta) \exp\{i \mathbf{f} \rho^2\} \exp\{-i 2\pi r \rho \cos(\vartheta - \varphi)\} \rho d\rho d\vartheta.$$

$$\exp\{i f \rho^2\} \rightarrow \exp\left\{i \left(\frac{f}{1 + \frac{f\lambda}{\pi R \sin^2 \alpha}}\right) \rho^2\right\} = \exp(i f' \rho^2) \left(\begin{array}{l} \text{compaction of distance} \\ \text{between axial intensity} \\ \text{zeros towards aperture} \end{array} \right).$$

$$\exp\{-i 2\pi r \rho \cos(\vartheta - \varphi)\} \rightarrow \exp\{-i 2\pi r' \rho \cos(\vartheta - \varphi)\} \text{ with}$$

$$r' = r \left(\frac{1}{1 + \frac{f\lambda}{\pi R \sin^2 \alpha}} \right) \left\{ 1 - \frac{\lambda^2}{4\pi^2 R^2 \sin^2 \alpha} \left[1 - \frac{2f\lambda}{\pi R \sin^2 \alpha} \right] r^2 \right\} \left(\begin{array}{l} \text{lateral expansion diffraction} \\ \text{pattern as a function of} \\ \text{distance from aperture} \end{array} \right).$$

Normalized amplitude correction factor :

$$\frac{1}{1 + \frac{f\lambda}{\pi R \sin^2 \alpha}}$$

Debye → Rayleigh:

region of applicability of the coordinate transformation

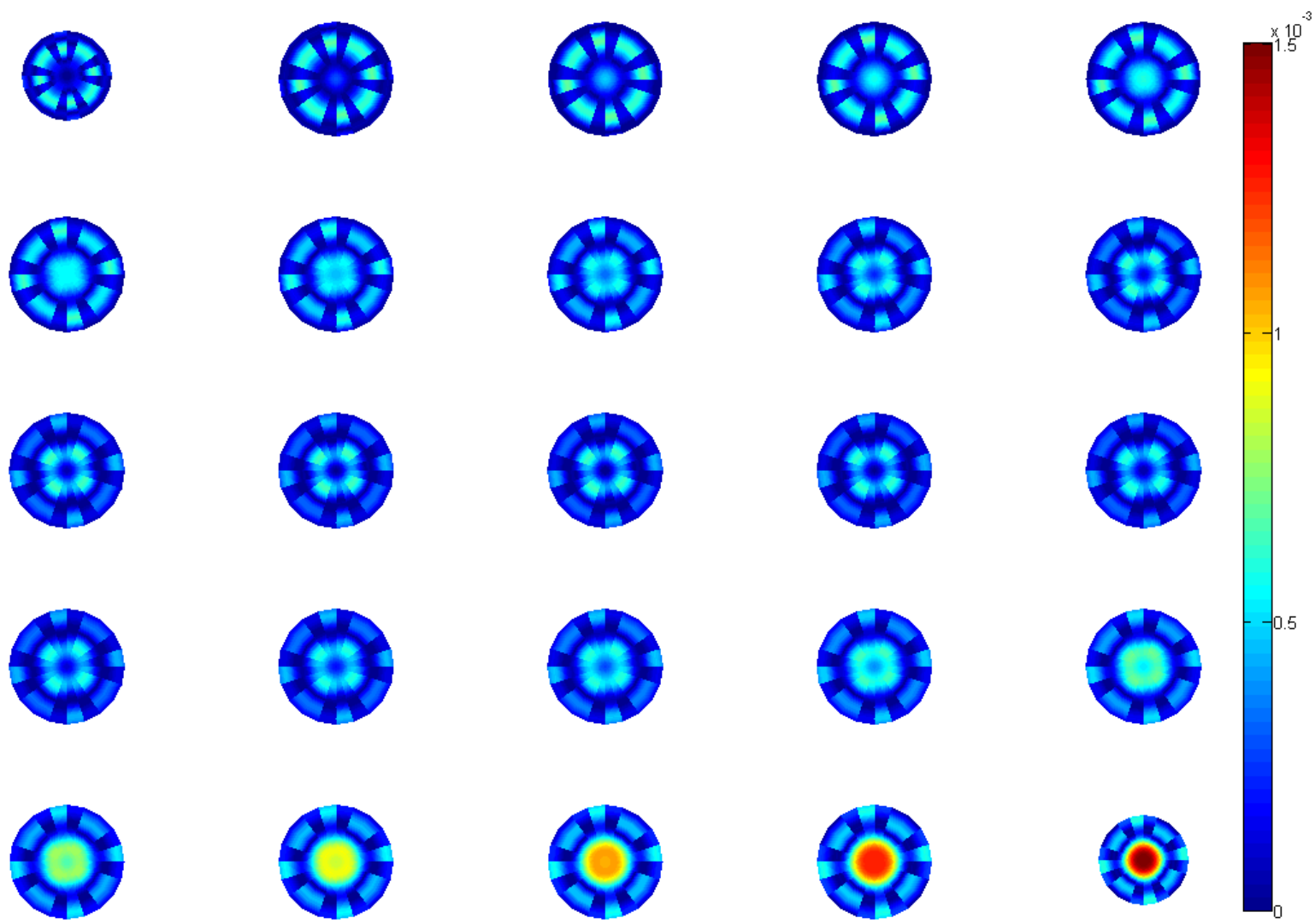
$$U_D(x, y, z) = \frac{-i \sin^2 \alpha}{\lambda} \iint_{\text{unit circle}} U_p(\rho, \vartheta) \exp\{i \mathbf{f} \rho^2\} \exp\{-i 2\pi \mathbf{r} \rho \cos(\vartheta - \varphi)\} \rho d\rho d\vartheta.$$

In adapting the Debye integral to the Rayleigh integral by an axial and lateral coordinate transformation ($r \rightarrow r'$, $f \rightarrow f'$), higher order terms in the exponential that are proportional to $r^2 \rho^2$ and $r \rho^3$ could not be included.

With the requirement that these exponential factors nowhere exceed the value of 0.1 radian, we find the following region of applicability of the coordinate transformation

- a) pupil diameter $2\rho_0 \geq 60\lambda$
- b) axial distance exit pupil to focus $R \geq 300\lambda$

Debye → Rayleigh: residual errors after transformation

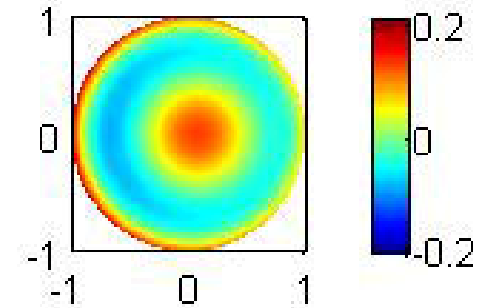
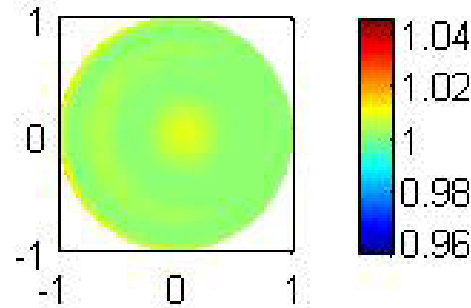


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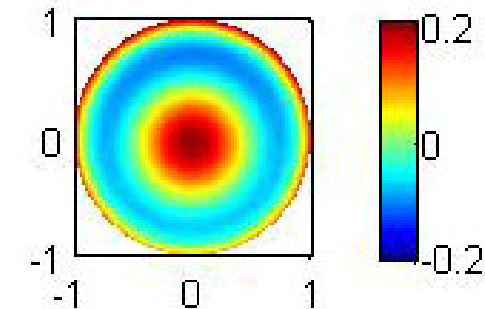
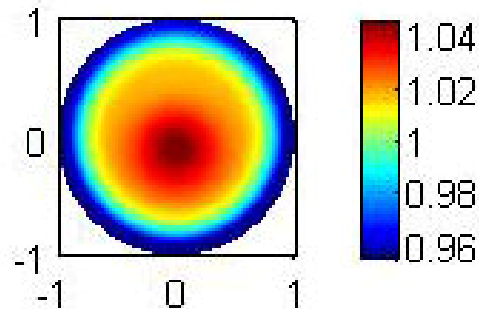
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Pupil function retrieval based on adapted Debye integral

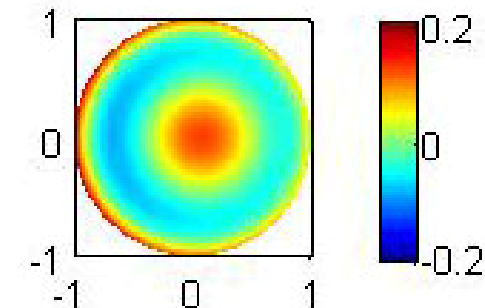
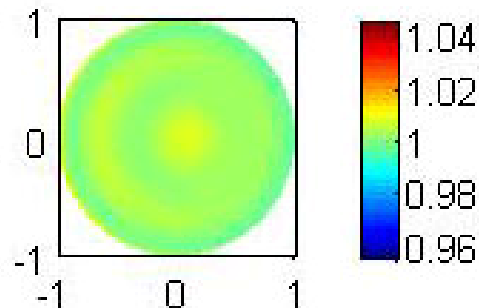
Input pupil function
(amplitude / phase)



Standard retrieval
(Debye integral)

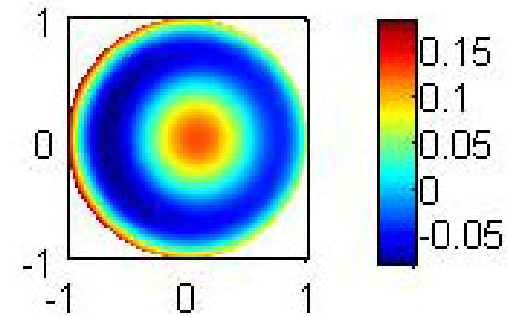
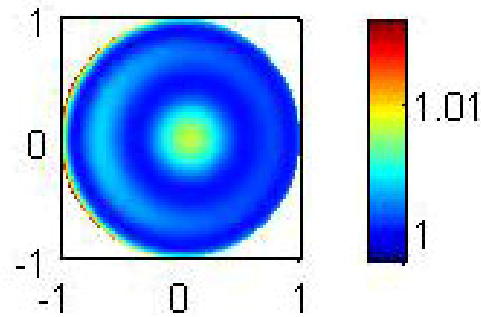


Retrieval using adapted
Debye integral

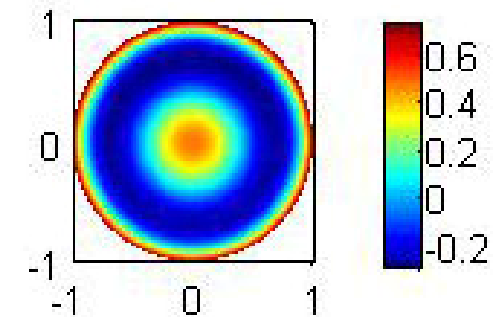
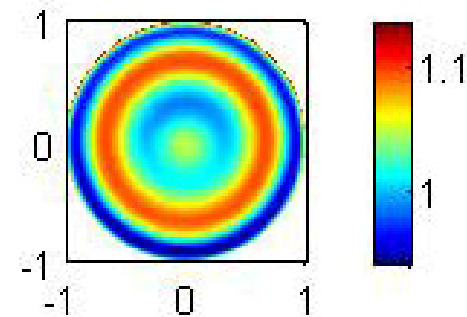


Microlens pupil function retrieval

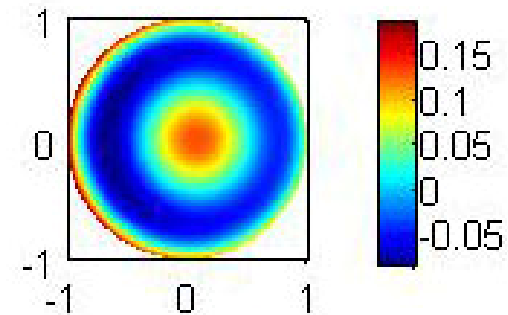
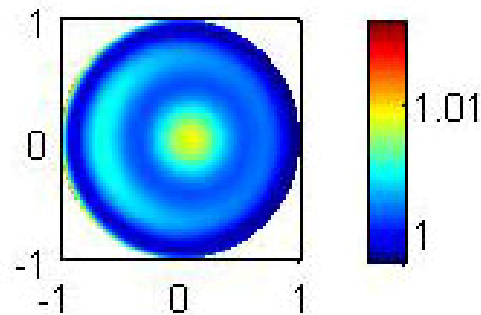
Microlens pupil function
(amplitude / phase)



Standard retrieval
(Debye integral)



Retrieval using adap-
ted Debye integral



Pupil function retrieval, numerical data

NA = 0.005, diameter pupil = 0.1 mm, $\lambda=266$ nm

Input pupil function (complex Zernike coefficients)

$$\begin{aligned}\beta_0^0 &= 1.000 & \beta_2^0 &= 0.000 & \beta_4^0 &= 0.1230i \\ \beta_3^1 &= 0.0300i & \beta_3^{-1} &= 0.0300i\end{aligned}$$

Standard retrieval with Debye integral

$$\begin{aligned}\beta_0^0 &= 0.9942 & \beta_2^0 &= -0.0481 - 0.0285i & \beta_4^0 &= -0.0184 + 0.1719i \\ \beta_3^1 &= -0.0284 + 0.0053i & \beta_3^{-1} &= 0.0284 - 0.0053i\end{aligned}$$

Retrieval with adapted Debye integral

$$\begin{aligned}\beta_0^0 &= 1.0001 & \beta_2^0 &= -0.0020 & \beta_4^0 &= -0.0025 + 0.1227i \\ \beta_3^1 &= 0.0299i & \beta_3^{-1} &= 0.0299i\end{aligned}$$

Microlens pupil function retrieval, numerical data

NA = 0.01, diameter pupil = 0.04 mm, $\lambda=400$ nm

Input pupil function (complex Zernike coefficients)

$$\begin{aligned}\beta_0^0 &= 1.000 & \beta_2^0 &= 0.000 & \beta_4^0 &= 0.1230i \\ \beta_3^1 &= 0.0300i & \beta_3^{-1} &= 0.0300i\end{aligned}$$

Standard retrieval with Debye integral

$$\begin{aligned}\beta_0^0 &= 0.9931 & \beta_2^0 &= -0.0516 + 0.1161i & \beta_4^0 &= -0.1119 + 0.5887i \\ \beta_3^1 &= -0.0244 + 0.0087i & \beta_3^{-1} &= -0.0244 + 0.0087i\end{aligned}$$

Retrieval with adapted Debye integral

$$\begin{aligned}\beta_0^0 &= 1.0001 & \beta_2^0 &= -0.0022 + 0.0003i & \beta_4^0 &= -0.0017 + 0.1259i \\ \beta_3^1 &= 0.0299i & \beta_3^{-1} &= 0.0299i\end{aligned}$$

Conclusion

- Small-NA imaging ($NA < 0.01$) introduces asymmetry around focus
- The ENZ pupil function retrieval method based on the Debye integral has to be adapted by an axial and lateral coordinate transformation that scales with the defocusing ($NA \leq 0.01$)
- The region of applicability of the adapted retrieval method is adequate for microscope objectives and microlenses provided the physical diameter of the lens aperture exceeds typically 50 wavelengths
- Numerical experiments showed the validity of the adapted ENZ retrieval method for small NA systems and microlenses