

Complex pupil function reconstruction at high numerical aperture using the extended Nijboer-Zernike diffraction theory

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ABSTRACT

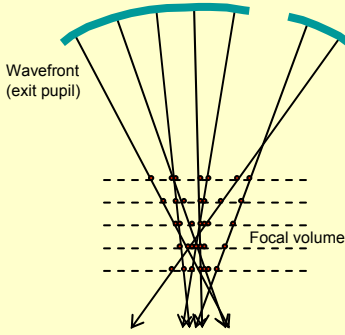
We have derived an analytical expression for the field components in the focal region of a high-numerical-aperture imaging system using the so-called Extended Nijboer-Zernike diffraction theory. It is shown that the transmission function, aberrations and polarization properties of an imaging system with high numerical aperture can be derived from the through-focus intensity map via an inversion process based on this analysis.

Problem definition:

How to retrieve optical system properties (*amplitude, phase and polarization* in the exit pupil) from *intensity* measurements through the *focal volume*?

1) Intuitive picture

(based on ray optics)



Ray density variation in the focal volume due to wavefront aberration and transmission variation

2) Scalar imaging

Analytic 'tool' derived from the Extended Nijboer-Zernike theory, first developed for the scalar imaging case ('point source')

$$U(r, \varphi; f) = \frac{1}{\pi} \int_0^1 \rho \exp(if\rho^2) \int_0^{2\pi} A(\rho, \vartheta) \exp[i\Phi(\rho, \vartheta)] \exp[i2\pi\rho \cos(\vartheta - \varphi)] d\vartheta d\rho$$

Introduction of Zernike polynomial expansion representing *amplitude and phase*:

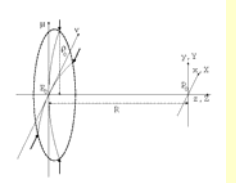
$$A(\rho, \vartheta) \exp[i\Phi(\rho, \vartheta)] = \sum_{n,m} \beta_n^m R_n^m \exp(im\vartheta)$$

Semi-analytic solution, see A.J.E.M. Janssen, J. Opt. Soc. Am **A19**, 849-857 (2002):

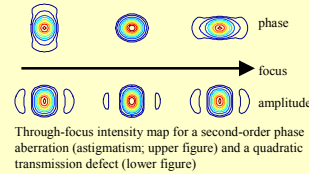
$$U(r, \varphi; f) = 2 \sum_{n,m} i^m \beta_n^m V_n^m(r, f) \cos m\varphi \quad V_n^m(r, f) = \int_0^1 \rho \exp(if\rho^2) R_n^m(\rho) J_m(2\pi\rho r) d\rho$$

$$V_n^m(r, f) = \exp(if) \sum_{l=1}^{\infty} (-2if)^{l-1} \sum_{j=0}^p v_j \frac{J_{m+l+2j}(v)}{l v^l}; \quad p = (n-m)/2, \quad v = 2\pi r$$

$$v_j = (-1)^j (m+l+2j) \binom{m+j+l-1}{l-1} \binom{j+l-1}{l-1} \binom{l-1}{p-j} / \binom{q+l+j}{l}; \quad q = (n+m)/2$$



Exit pupil and focal volume: co-ordinates



Through-focus intensity map for a second-order phase aberration (astigmatism; upper figure) and a quadratic transmission defect (lower figure)

3) Vectorial imaging

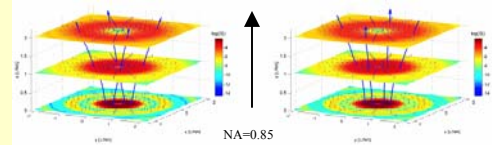
Complication in a high NA imaging system: *vectorial* calculation of the aberrated intensity distribution is needed in the focal volume!

$$\text{Electric field density in the focal volume is given by } w_E = \frac{\epsilon_0}{4} n^2 |E|^2 = \frac{\epsilon_0}{4} n^2 \{E_x E_x^* + E_y E_y^* + E_z E_z^*\}$$

The *electric field vector* produced by a point source illuminating an aberrated optical system (complex pupil function given by two sets of Zernike coefficients $\beta_n^{m,x}, \beta_n^{m,y}$) is given by

$$\vec{E}^x(r, \varphi; f) = -i\gamma s_0^2 \exp\left(\frac{-if}{1-\sqrt{1-s_0^2}}\right) \sum_{n,m} i^m \beta_n^{m,x} \exp(im\varphi) \begin{pmatrix} V_{n,0}^m + \frac{s_0^2}{2} V_{n,2}^m \exp(2i\varphi) + \frac{s_0^2}{2} V_{n,-2}^m \exp(-2i\varphi) \\ -\frac{is_0^2}{2} V_{n,2}^m \exp(2i\varphi) + \frac{is_0^2}{2} V_{n,-2}^m \exp(-2i\varphi) \\ -is_0 V_{n,1}^m \exp(i\varphi) + is_0 V_{n,-1}^m \exp(-i\varphi) \end{pmatrix}$$

see: J.J.M. Braat, P. Dirksen, A.J.E.M. Janssen, A.S. van de Nes, J. Opt. Soc. Am. **A20**, 2281-2292 (2003)



Vectorial *forward* calculation using the semi-analytic Extended Nijboer-Zernike theory. Energy flow (arrows) and intensity distribution (colour coded) in the focal region; Left: circularly polarized Right: circularly polarized + orbital angular momentum

Energy density in the focal region

$$\beta_n^{m,x} = a\beta_n^m, \quad \beta_n^{m,y} = b\beta_n^m; \quad (|a|^2 + |b|^2 = 1 \text{ for normalisation purposes})$$

$$w_E(r, \varphi; f) = \frac{\epsilon_0 n^2 s_0^2}{4} \begin{pmatrix} G_{00}(\beta, \beta) + s_0^2 \{ [|a|^2 - |b|^2] \text{Re}\{G_{0,2}(\beta, \beta)\} - 2 \text{Re}(ab^*) \text{Im}\{G_{0,2}(\beta, \beta)\} \} + s_0^2 \{ [|a|^2 - |b|^2] \text{Re}\{G_{0,-2}(\beta, \beta)\} + 2 \text{Re}(ab^*) \text{Im}\{G_{0,-2}(\beta, \beta)\} \} + \frac{s_0^4}{2} \{ [-2 \text{Im}(ab^*) \} G_{2,2}(\beta, \beta) + [+2 \text{Im}(ab^*) \} G_{2,-2}(\beta, \beta) \} + s_0^2 \{ [-2 \text{Im}(ab^*) \} G_{1,1}(\beta, \beta) + [+2 \text{Im}(ab^*) \} G_{1,-1}(\beta, \beta) \} + -2s_0^2 \{ [|a|^2 - |b|^2] \text{Re}\{G_{1,-1}(\beta, \beta)\} + 2 \text{Re}(ab^*) \text{Im}\{G_{1,-1}(\beta, \beta)\} \} \end{pmatrix}$$

$$G_{kl}(\alpha, \beta) =$$

$$\sum_{n,m} i^m \alpha_n^m V_{n,k}^m \exp(im\varphi) \exp(ik\varphi) \times \sum_{n',m'} i^{-m'} \beta_{n'}^{m'} V_{n',l}^{m'*} \exp(-im'\varphi) \exp(-il\varphi) =$$

$$\sum_{n',m'} \exp\{i(m-m')\pi/2\} \exp\{i(m-m'+k-l)\varphi\} \alpha_n^m \beta_{n'}^{m'*} V_{n,k}^m(r, f) V_{n',l}^{m'*}(r, f)$$

Small β_n^m -approximation (dominating β_0^0 -coefficient)

$$G_{kl}(\alpha, \beta) = e^{i(k-l)\varphi} \sum_{\nu=0}^{\nu_{\max}(\nu)} \sum_{\mu=-\mu_{\max}(\nu)}^{+\mu_{\max}(\nu)} \left\{ e^{-i\mu\pi/2} e^{-i\mu\varphi} \alpha_0^0 \beta_{\nu,k}^{\mu*} V_{0,k}^0(r, f) V_{\nu,l}^{\mu*}(r, f) + (1 - \epsilon_{\nu\mu}) e^{+i\mu\pi/2} e^{+i\mu\varphi} \alpha_{\nu}^{\mu} \beta_0^0 V_{\nu,k}^{\mu}(r, f) V_{0,l}^0(r, f) \right\}$$

Conclusions

- 1) A parametric analytic expression has been found for the electric field and the energy density in the focal volume of an optical system including pupil aberrations, pupil transmission variations and spatially varying birefringence of the imaging system (small source point, high NA)
- 2) Complex Zernike coefficients (two sets) are suitable to describe the generalized state of polarization in the exit pupil of the optical system
- 3) The *inverse* problem can be solved using a through-focus intensity map (energy density function) to solve a system of linearized equations in the two unknown sets of Zernike coefficients
- 4) The extended Nijboer-Zernike analysis offers an eigenfunction expansion for the vectorial propagation operator from exit pupil to image volume. This property can be exploited for the fast *forward* calculation of the image intensity in the focal volume