

POLARISATION-ABERRATION RETRIEVAL FOR HIGH-NA SYSTEMS USING THE EXTENDED NIJBOER-ZERNIKE DIFFRACTION THEORY

Joseph J.M. Braat¹, Peter Dirksen³, Augustus J.E.M. Janssen², Arthur S. van de Nes¹

¹ Delft University of Technology, Delft, ² Philips Research Laboratories, Eindhoven, The Netherlands; ³ Philips Research Laboratories, Leuven, Belgium.

ABSTRACT

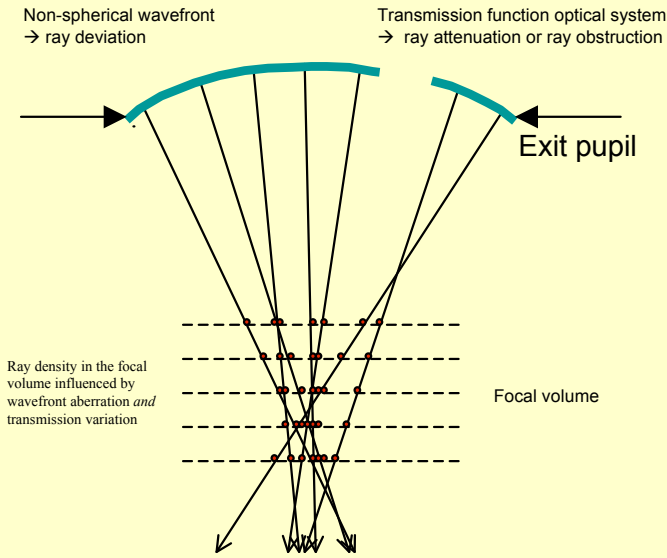
We have derived analytical expressions for the field components in the focal region of a high-numerical-aperture imaging system using the so-called Extended Nijboer-Zernike diffraction theory. It is shown that the transmission function, aberrations and polarisation properties of an imaging system with high numerical aperture can be derived from the through-focus intensity map via an inversion process based on this analysis.

Problem definition:

How to retrieve optical system properties (*amplitude, phase and polarisation* in the exit pupil) from *intensity* measurements through the *focal volume*?

1) Intuitive picture, based on ray optics →

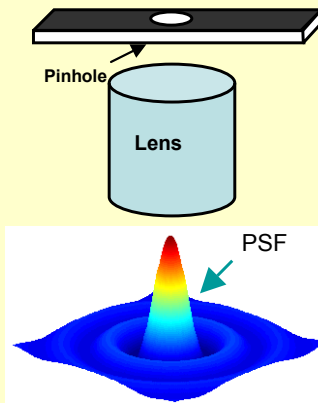
change in ray direction (wavefront aberration) and ray attenuation determine ray density (intensity) in focal volume !



2) More rigorous picture is based on scalar diffraction theory →

Huygens-Fresnel diffraction integral for light propagation from exit pupil → image plane.

In the presence of aberrations: theory of Nijboer-Zernike (1942)



Extension :

from source →

to exit pupil →

to focal volume :

Extended Nijboer-Zernike theory for through-focus point-spread function

For the Bessel series solution, see:

A.J.E.M. Janssen, J. Opt. Soc. Am **A19**, 849-857 (2002)

Basic diffraction integral with defocus:
$$U(r, \varphi; f) = \frac{1}{\pi} \int_0^1 \rho \exp(ik\rho^2) \int_0^{2\pi} A(\rho, \vartheta) \exp\{i\Phi(\rho, \vartheta)\} \exp\{i2\pi r \rho \cos(\vartheta - \varphi)\} d\vartheta d\rho$$

Introduction of Zernike polynomial expansion representing **amplitude and phase**:
$$A(\rho, \vartheta) \exp\{i\Phi(\rho, \vartheta)\} = \sum_{n,m} \beta_n^m R_n^{(m)} \exp(im\vartheta)$$

Bessel series solution:
$$U(r, \varphi; f) = 2 \sum_{n,m} i^m \beta_n^m V_n^{(m)}(r, f) \exp(im\varphi) \quad \text{with:} \quad V_n^{(m)}(r; f) = \exp(ikf) \sum_{l=0}^{\infty} \left(\frac{-if}{\pi r}\right)^l \sum_{j=0}^l u_{lj} \frac{J_{|m|+2j+l+1}(2\pi r)}{2\pi r}$$

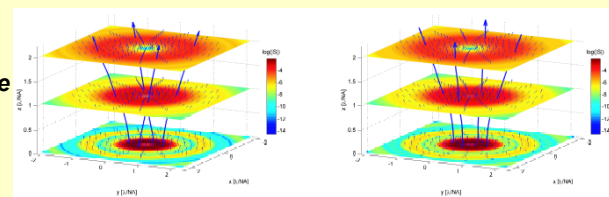
3) Vector diffraction theory for high-NA focused beams (forward) →

ENZ-theory for complex exit pupil function with due account of

a) radiometric effect b) high-NA defocusing factor c) polarisation state

Electric field density in the focal volume is given by
$$w_E = \frac{\epsilon_0}{4} n^2 |E|^2 = \frac{\epsilon_0}{4} n^2 \{E_1 E_1^* + E_2 E_2^* + E_3 E_3^*\}$$
 with explicit analytic expressions available for electric field vector components in the focal volume.

See: J.J.M. Braat, P. Dirksen, A.J.E.M. Janssen, A.S. van de Nes, J. Opt. Soc. Am. **A20**, 2281-2292 (2003)



Vectorial **forward** calculation using the Extended Nijboer-Zernike theory. Energy flow (arrows) and intensity distribution (colour-coded) in the focal region. Left-hand figure: circularly polarised Right-hand figure: circularly polarised + orbital angular momentum

4) Polarisation-aberration retrieval using ENZ-theory at high NA:

'backward' calculation from energy density in focal volume leads to

→ complex lens function + polarisation effects (birefringence)

State of polarisation in the exit pupil depends on :

- a) geometrical lens properties (NA, transmission, aberration),
- b) birefringence ('scrambling' of polarisation state)

Description of state of polarisation via Jones matrix :

$$\begin{pmatrix} E_{x,j} \\ E_{y,j} \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} a_j \\ b_j \end{pmatrix} \quad \text{with} \quad \begin{pmatrix} a_j \\ b_j \end{pmatrix} \text{ the incident polarisation.}$$

For a pure phase birefringence, the Jones matrix reduces to :

$$\begin{pmatrix} E_{x,j} \\ E_{y,j} \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} \\ -m_{12}^* & m_{11}^* \end{pmatrix} \begin{pmatrix} a_j \\ b_j \end{pmatrix} \quad \text{with} \quad |m_{11}|^2 + |m_{12}|^2 = 1$$

Conclusion:

four 'inverse' operations are sufficient for retrieval of the 'polarisation-aberrations' of an optical imaging system

Detailed information about the ENZ-theory and its applications in optical aberration theory, lithography and lens metrology can be found at the website:

<http://www.nijboerzernike.nl>